ISSN 2229-5518

Algorithm of evaluation of the unstable equilibrium state of dynamically tuned gravimeter

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Abstract— The article describes the development and investigation of the algorithm of valuation of the dynamically tuned gravimeter state using the least-squares method on the basis of information about the movement of sensors regarding the unstable equilibrium; evaluation errors caused by the inadequacy of the accepted approximation of the model and the real signal, and the errors caused by the kinematic nonlinearity have been also studied and described.

Index Terms— gravimetric aviation system, dynamically tuned gravimeter, output signal, evaluation errors, algorithm.

1 INTRODUCTION

The information about the Earth's gravitational field anomalies is used in inertial navigation and in the search of mineral resources. Those gravimeters (among the known ones) [1-2] that are used in aviation gravimetric systems have the lowest accuracy, since they are the most disadvantaged in terms of measuring and eliminating external factors that influence the measuring signal of the gravimeter. Therefore, increasing the accuracy and speed of the dynamically tuned gravimeter of the gravimetric aviation system (GAS) with the automatic processing of information is conditioned by the need of establishing the effective and easy-to-implement algorithms of valuation of the dynamically tuned gravimeter (DTG) state.

In the analyzed literature [3-8] the modifications of the optimal and suboptimal algorithms of discrete signal filtering are investigated, but the studies of the DTG accuracy [3-8] do not take into account the impact of gyrogravimeter errors caused by nonlinear distortions of gyroscope motion path, by inequality of index of precession damping vibrations to zero due to the influence of viscous friction, by nonisochism of precession fluctuations, by the mismatch of precession vibrations angular frequency used in the evaluation algorithms to the gyroscope precession fluctuations frequency, by the barriers that distort the gyroscope motion law. The impact of these errors, if they are not taken into account, significantly influences the output signal of DTG and reduces the measurement accuracy of Earth's gravitational field anomalies.

Since the task of optimal filtration is the task of the state evaluation in stochastic terms, we will use the term "state evaluation" in the future . Taking into account the interconnection of the methods of least squares (MLS) and optimal Kalman filter (KF), we will consider that MLS is the simplest approach to the state evaluation, and KF is the only optimal evaluation algorithm. Optimal Kalman filter was investigated in [9]. The rest of the evaluation algorithms are intermediate between MLS and KF, so it is reasonable to further investigate only these two algorithms.

The aim of this paper is to develop DTG state evaluation algorithm under the conditions of orientation of its sensitivity axis to the north and to the south by the method of least squares. The achievement of this goal will allow the improvement of the accuracy and speed of DTG GAS measurement by eliminating previously uncompensated errors.

To achieve this goal it is necessary to solve the following problems:

1) to develop DTG state evaluation algorithm under the conditions of orientation of its sensitivity axis to the north and to the south (stable and unstable equilibrium) on the basis of the method of least squares;

2) to investigate evaluation errors, to determine their dependence on the perturbation parameters, the initial conditions of dynamically tuned gravimeter motion state, sensitive element (SE);

3) to analyze the evaluation errors caused by the evaluation algorithms sensitivity to the discrepancies of the accepted model and real output signal of DTG;

4) to investigate the influence of distortion of DTG motion law seen as the typical distortions and measurement channel angle sensor (AS) noise.

To solve this problem, DTG [10, 11] was applied . Output signal of dynamically tuned gravimeter, which is taken from the AS is fed into the computer . The computer determines the value of the errors using the algorithm described in this article and provides automatic compensation of these errors in the output signal of the gyrogravimeter. Then the adjusted signal of the gyrogravimeter along with the signals from other subsystems of GAS is used to calculate Δg.

Main part. As in [11], sensing element motion observed by the angle sensor is represented by the function

$$\alpha(t) = R^S + \alpha_1^*(t) + \varepsilon(t)\,. \tag{1}$$

where R^S is a true angle between AS zero and the direction to the south; $\alpha_1^*(t)$ is current angular position of SE relative to

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the direction to the south, which is determined as the solution of equation (1) in the case of replacement $\alpha_1(t) = \pi + \alpha_1^*(t)$. For this case the differential equation (1) can be written as

$$\ddot{\alpha}_1^*(t) + 2\xi_1\dot{\alpha}_1^* - \omega_0^2 \sin\alpha_1^*(t) = 0 . \quad (2)$$

We will omit index "*" for the ease of writing. When $\sin\alpha \approx \alpha$, the solution of equation (2) is described by the expression

$$\alpha_1(t) = e^{-\xi_1 t}(c_{10} sh\omega_0 t + c_{20} ch\omega_0 t) , \quad (3)$$

where c_{10}, c_{20} are determined by the initial conditions at t=0. Based on the expressions (1) and (2), when $\xi_1 = 0$, the SE motion model looks like

$$\alpha(t) = \hat{R}^S + \hat{c}_1 sh\omega_0 t + \hat{c}_2 ch\omega_0 t , \quad (4)$$

where $\hat{R}^S$ is the value of the angle between AS zero and the state of unstable equilibrium; $\hat{c}_1, \hat{c}_2$ is the values of c_{10}, c_{20} respectively.

To develop the evaluation algorithm we generate the quadratic functional

$$F_S = \sum_{i=1}^{n}(\hat{R}^S + \hat{c}_1 sh\omega_0 t_i + \hat{c}_2 ch\omega_0 t_i - \alpha_i)^2 . \quad (5)$$

The system of algebraic equations

$$\begin{cases} \hat{R}^S \sum_{i=1}^{n} 1 + \hat{c}_1 \sum_{i=1}^{n} sh\omega_0 t_i + \hat{c}_2 \sum_{i=1}^{n} ch\omega_0 t_i = \sum_{i=1}^{n} \alpha_i , \\ \hat{R}^S \sum_{i=1}^{n} sh\omega_0 t_i + \hat{c}_1 \sum_{i=1}^{n} sh^2\omega_0 t_i + \hat{c}_2 \sum_{i=1}^{n} sh\omega_0 t_i = \sum_{i=1}^{n} \alpha_i sh\omega_0 t_i , \\ \hat{R}^S \sum_{i=1}^{n} ch\omega_0 t_i + \hat{c}_1 \sum_{i=1}^{n} sh\omega_0 t_i ch\omega_0 t_i + \hat{c}_2 \sum_{i=1}^{n} ch^2\omega_0 t_i = \sum_{i=1}^{n} \alpha_i ch\omega_0 t_i . \end{cases} \quad (6)$$

meets the terms of the minimum functional.

System observation analysis (6), conducted the same way as in [9], shows that observation (solvability) of the system under study is provided on condition that

$$\lambda = \omega_0 \Delta t \neq 0; \;\; n \geq 3 . \quad (7)$$

After replacing of hyperbolic functions on the exponential, system (6) can be represented by a matrix equation

$$c_{\lambda,n}^S \hat{x}_S = z_{\lambda,n}^S , \quad (8)$$

where

$$c_{\lambda,n}^S = \begin{vmatrix} n & \frac{1-e^{\lambda n}}{1-e^{\lambda}} & e^{\lambda(1-n)}\frac{1-e^{\lambda n}}{1-e^{\lambda}} \\ \frac{1-e^{\lambda n}}{1-e^{\lambda}} & \frac{1-e^{2\lambda n}}{1-e^{2\lambda}} & n \\ e^{\lambda(1-n)}\frac{1-e^{\lambda n}}{1-e^{\lambda}} & n & e^{2\lambda(1-n)}\frac{1-e^{2\lambda n}}{1-e^{2\lambda}} \end{vmatrix},$$

$$\hat{x}_S = [\hat{R}^S \hat{c}_1 \hat{c}_2]^T, \quad z_{\lambda,n}^S = [S_1^S S_2^S S_3^S]^T = [\sum_{i=1}^{n}\alpha_i \sum_{i=1}^{n}\alpha_i e^{\lambda(i-1)} \sum_{i=1}^{n}\alpha_i e^{-\lambda(i-1)}]^T .$$

Solving the system (8) by Kramer method, we obtain an expression for the state vector elements that are measured in the form of

$$\hat{R}^S = \frac{A_{11}^S S_1^S + A_{22}^S S_2^S + A_{31}^S S_3^S}{\det c_{\lambda,n}^S};$$

$$\hat{c}_1 = \frac{A_{12}^S S_1^S + A_{22}^S S_2^S + A_{31}^S S_3^S}{\det c_{\lambda,n}^S};$$

$$\hat{c}_2 = \frac{A_{13}^S S_1^S + A_{23}^S S_2^S + A_{33}^S S_3^S}{\det c_{\lambda,n}^S},$$

where A_{ij} are algebraic complements, element c_{ij}^S $(i, j = \overline{1,3})$ of matrix $c_{\lambda,n}^S$; $\det c_{\lambda,n}^S$ is the determinant of matrix. Revealing more detailed the expression $\hat{R}^S$, we obtain

$$\hat{R}_S = \left\{ S_1^S \left[\frac{1+e^{\lambda n}}{1+e^{\lambda}} + ne^{\lambda(n-1)}\frac{1-e^{\lambda}}{1-e^{\lambda n}} \right] - S_2^S - S_3^S e^{\lambda(n-1)} \right\} \times$$
$$\times \left\{ n\left[\frac{1+e^{\lambda n}}{1+e^{\lambda}} + ne^{\lambda(n-1)}\frac{1-e^{\lambda}}{1-e^{\lambda n}} \right] - 2\frac{1-e^{\lambda m}}{1-e^{\lambda}} \right\}^{-1} . \quad (9)$$

Thus, the DTG unstable equilibrium state evaluation algorithm can be represented either by the system solution (8) or by calculation expression (9).

We will study the evaluation algorithm errors. To do this, we will find the solutions of differential equation (2) by the method of successive approximations when $\sin\alpha_1 \approx \alpha_1 - \frac{\alpha_1^3}{3!}$. At first, we introduce the replacement $x = \alpha_1$.

We write the equation

$$\varepsilon(t) = k_\partial t , \quad (10)$$

where k_∂ - slope drift, taking into account the remarks in the first approximation :

$$\ddot{x}_1 + 2\xi\dot{x}_1 - \omega_0^2 x = 0 . \quad (11)$$

The solution of the equation of the first approximation has the form

$$x_1 = (c_{10}e^{\omega_{10} t} + c_2 e^{-\omega_0 t})e^{-\xi t} . \quad (12)$$

We write the equation of the second approximation

$$\ddot{x}_2 + 2\xi\dot{x}_2 - \omega_0 x_2 = -\frac{x_1^3}{6} . \quad (13)$$

Taking into account the smallness of the second approximation in equation (13), set ξ=0; considering the expression (12) we find the solution of the second approximation from the equation

$$\ddot{x}_2 - x_1\omega_0^2 = -\frac{1}{6}(c_{10}^3 e^{3\omega_0 t} + 3c_{10}^2 c_{20} e^{\omega_0 t} + 3c_{10}c_{20}^2 e^{-\omega_0 t} + c_{20}^3 e^{-3\omega_0 t}) . \quad (14)$$

From equation (14) we obtain the second approximation solution

$$x_2 = -\frac{1}{48}(c_{10}^3 e^{3\omega_0 t} + c_{20}^3 e^{-3\omega_0 t}) - \frac{c_{10}c_{20}\omega_0 t}{4}(c_{10}e^{\omega_0 t} - c_{20}e^{\omega_0 t}) . \quad (15)$$

Taking into account the second approximation (expression (15) and Maclaurin series decomposition in the parameter ξ_1, find the law of SE motion nearby the state of unstable equilibrium

$$\alpha(t)=R_0^S+c_{10}e^{\omega_0 t}+c_{20}e^{-\omega_0 t}-\xi t(c_{10}e^{\omega_0 t}+c_{20}e^{-\omega_0 t})- \quad (16)$$
$$-\frac{1}{48}(c^3{}_{10}e^{3\omega_0 t}+c^3{}_{20}e^{-3\omega_0 t})-\frac{c_{10}c_{20}\omega_0 t}{4}(c_{10}e^{\omega_0 t}-c_{20}e^{-\omega_0 t})+\varepsilon(t).$$

On the other hand, having expressed $sh\omega_0 t,\ ch\omega_0 t$ through exponential functions of $e^{\omega_0 t}, e^{-\omega_0 t}$ type and having put the model functions (4) in Taylor series according to the parameters $\hat R^S,\hat c_1,\hat c_2,\omega_0$ in the neighborhood of the point $\hat R_0^S,\hat c_{10},\hat c_{20},\hat\omega_0$, keeping the first two expansion terms, we obtain

$$\alpha(t)=R_0^S+\Delta R^S+\hat c_{10}e^{\omega_0 t}+\hat c_{20}e^{-\omega_0 t}+\Delta\hat c_1 e^{\omega_0 t}+\Delta\hat c_2 e^{-\omega_0 t}+ \quad (17)$$
$$+\Delta\omega t(\hat c_{10}e^{\omega_0 t}-\hat c_{20}e^{-\omega_0 t}),$$

where $\Delta\hat R^S,\Delta\hat c_1,\Delta\hat c_2$ are state evaluation errors; $\Delta\hat R^S=\hat R^S-\hat R_0^S,\Delta\hat c_1=\hat c_1-c_{10},\Delta\hat c_2=\hat c_2-c_{20}$; $\Delta\omega=\omega-\omega_0$ is the error of ω_0.

Putting expressions (16) and (17) into the functional (5), we obtain

$$F_S=\sum_{i=1}^{n}[\Delta\hat R^s+\Delta\hat c_1 e^{\omega_0 t_i}+\Delta\hat c_2 e^{-\omega_0 t_i}-\alpha_S^0(t_i)]^2, \quad (18)$$

where

$$\alpha_S^0(t_i)=-\Delta\omega t_i(c_{10}e^{\omega_0 t_i}-c_{20}e^{-\omega_0 t_i})-\xi_1 t_i(c_{10}e^{\omega_0 t_i}+c_{20}e^{-\omega_0 t_i})- \quad (19)$$
$$-\frac{1}{48}(c_{10}^3e^{3\omega_0 t_i}+c_{20}^3e^{-3\omega_0 t_i})-\frac{c_{10}c_{20}\omega_0 t_i}{4}(c_{10}e^{\omega_0 t_i}-c_{20}e^{-\omega_0 t_i})+\varepsilon(t_i).$$

Terms of achieving the functional minimum are equivalent to the system of equations

$$c_{\lambda_0,n}^S\hat\delta^S=\bar f^S, \quad (20)$$

where

$$\hat\delta^S=[\Delta\hat R^S\Delta\hat c_1\Delta\hat c_2]^T,\quad \hat f^S=[f_1^S f_2^S f_3^S]^T=\begin{bmatrix}\sum_{i=1}^{n}\alpha_S^0(t_i)\sum_{i=1}^{n}\alpha_S^0(t_i)e^{\lambda_0(i-1)}\times\\ \times\sum_{i=1}^{n}\alpha_S^0(t_i)e^{-\lambda_0(i-1)}\end{bmatrix}^T,$$

$$c_{\lambda_0,n}^S=c_{\lambda\ ,n}^S\Big|_{\lambda=\lambda_0},\quad \lambda_0=\omega_0\Delta t.$$

Put the expression (9) into the system (20) and find the evaluation error

$$\Delta\hat R^S=\left\{f_1^S\left[\frac{1+e^{\lambda_0 u}}{1+e^{\lambda_0}}+ne^{\lambda_0(n-1)}\frac{1-e^{\lambda_0}}{1-e^{\lambda_0 n}}\right]-f_2^S-f_3^S e^{\lambda_0(n-1)}\right\}\times \quad (21)$$
$$\times\left\{n\left[\frac{1+e^{\lambda_0 n}}{1+e^{\lambda_0}}+ne^{\lambda_0(n-1)}\frac{1-e^{\lambda_0}}{1-e^{\lambda_0 n}}\right]-2\frac{1-e^{\lambda_0 n}}{1-e^{\lambda_0}}\right\}^{-1}.$$

Having made the boundary transition at $\Delta t\to 0$ and $T_H=const$, we obtain an expression of the evaluation error

$$\Delta\hat R^S=\sum_{j=1}^{S}\Delta R_j^S, \quad (22)$$

where

$$\Delta\hat R^S{}_1=-\frac{1}{48}(c_{10}^3e^{\frac{3u}{2}}+c_{20}^3e^{-\frac{3u}{2}})\cdot l_2(u), \quad (23)$$

$$\Delta\hat R_2^S=-\frac{\xi_1}{\omega_0}l_1(u)(c_{10}e^{\frac u2}-c_{20}e^{-\frac u2}), \quad (24)$$

$$\Delta\hat R_3^S=-\frac{c_{10}c_{20}}{4}(c_{10}e^{\frac u2}+c_{20}e^{-\frac u2})\cdot l_1(u), \quad (25)$$

$$\Delta\hat R_4^S=-\frac{\Delta\omega}{\omega_0}l_1(u)(c_{10}e^{\frac u2}+c_{20}e^{-\frac u2}), \quad (26)$$

$$\Delta\hat R_5^S=\omega_0\Delta^{-1}\int_0^T\left\{\left[\frac12(e^u+1)+\frac{ue^u}{e^u-1}\right]-e^{\omega_0 t}-e^u e^{-\omega_0 t}\right\}\varepsilon(t)dt, \quad (27)$$

$$l_1(u)=e^{\frac u2}\Delta^{-1}\left[\frac{ue^u}{2}\left(\frac{u(e^u+1)}{e^u-1}-1\right)-\frac{e^{2u}-1}{4}\right], \quad (28)$$

$$l_2(u)=\frac13 e^{-\frac{3u}{2}}\Delta^{-1}\left[ue^u(e^{2u}+e^u+1)-\frac14(e^{2u}-1)(e^{2u}+4e^u+1)\right], \quad (29)$$

$$\Delta=u\left(\frac{e^u+1}{2}+\frac{ue^u}{e^u-1}\right)-2(e^u-1), u=\omega_0 T_c,$$

$\Delta\hat R_1^S$ is the component of the evaluation error caused in the law of motion of SE by the item $-\frac{1}{48}(c_{10}^3e^{3\omega_0 t}+c_{20}^3e^{-3\omega_0 t})$; $\Delta\hat R_2^S$ is the component of the evaluation error caused by neglecting the decrement of attenuation in the model (4); $\Delta\hat R_3^S$ is the component of the evaluation error caused in the law of motion of SE by the item $-\frac14 c_{10}c_{20}\omega_{10}t(c_{10}e^{\omega_0 t}-c_{20}e^{-\omega_0 t})$; $\Delta\hat R_4^S$ is the component of the evaluation error caused by the error of setting the parameter ω_0; $\Delta\hat R_5^S$ is the component of the evaluation error caused by the distortions of the observed law of motion of SE .

We will analyze the components of the evaluation error $\Delta\hat R_j^S(j=1{,}5)$.

As we can see from the obtained analytical expressions of these components of the error, their values depend on the parameters $\frac{\Delta\omega}{\omega_0},\frac{\xi_1}{\omega_0}$, the initial conditions $\alpha_0,\dot\alpha_0$, and the information surveillance time. To analyze the maximums of the components $\Delta\hat R_j^S$ we define a zone of initial conditions on the phase plane , located inside the circle of radius A, that is

$$\alpha_0^2+\left(\frac{\dot\alpha_0}{\omega}\right)^2\le A^2. \quad (30)$$

We use the polar coordinate system on the phase plane $O\alpha\dot\alpha$ (Fig. 1). We combine plus and polar axis with O start and abscissa axis $O\alpha$ of the rectangular coordinate system.

ISSN 2229-5518

We denote ρ and φ as polar radius and polar angle respectively, and the equation is

$$\alpha_0 = \rho\cos\varphi,\ \ \frac{\dot{\alpha}}{\omega} = \rho\sin\varphi\,. \tag{31}$$

Then the constants of integration c_{10}, c_{20} are expressed with the use of ρ and φ by such relations

$$c_{10} = \frac{\rho}{\sqrt{2}}\sin(\varphi + \frac{\pi}{4}),\ c_{20} = \frac{\rho}{\sqrt{2}}\cos(\varphi + \frac{\pi}{4})\,. \tag{32}$$

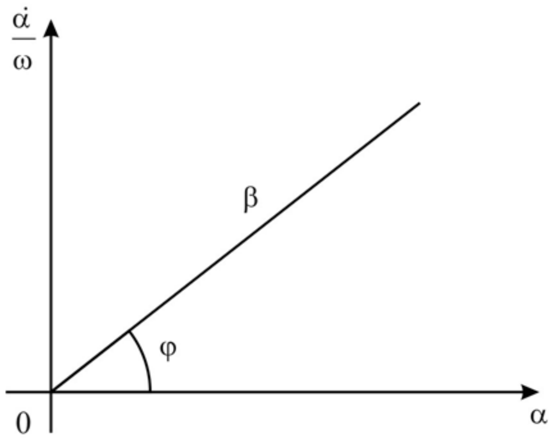

Fig. 1. Polar coordinate system

Taking into account (32), the expressions (23) ... (26) are written as .

$$\Delta R_1^S = -\frac{\rho^3}{96\sqrt{2}} l_2(u)\left[e^{\frac{3u}{2}}\sin^3(\varphi + \frac{\pi}{4}) + e^{-\frac{3u}{2}}\cos^3(\varphi + \frac{\pi}{4})\right], \tag{33}$$

$$\Delta\hat{R}_2^S = -\frac{\xi_1}{\omega_0}\rho l_1(u)\sqrt{chu}\sin(\varphi + \frac{\pi}{4} - arctge^{-u}), \tag{34}$$

$$\Delta\hat{R}_3^S = -\frac{\rho}{16} l_1(u)\sqrt{chu}\sin 2(\varphi + \frac{\pi}{4})\sin(\varphi + \frac{\pi}{4} + arctge^{-u}), \tag{35}$$

$$\Delta\hat{R}_4^S = -\frac{\Delta\omega}{\omega}\rho l_1(u)\sqrt{chu}\sin(\varphi + \frac{\pi}{4} + arctge^{-u})\,. \tag{36}$$

Based on the components of expressions (33) ... (36), we obtain the maximum values of evaluation error if ρ=A:

$$\begin{aligned}
\max(\Delta\hat{R}_1^S) &= \frac{A^3}{192} l_2(u) e^{\frac{3u}{2}}, \\
\max(\Delta\hat{R}_2^S) &= -\frac{|\xi_0|}{\omega_0} A l_1(u)\sqrt{chu}, \\
\max(\Delta\hat{R}_3^S) &= -\frac{A^3}{16} l_1(u)\sqrt{chu}\cdot m(u), \\
\max(\Delta\hat{R}_4^S) &= -\frac{\Delta\omega}{\omega_0} A l_1(u)\sqrt{chu},
\end{aligned} \tag{37}$$

where $m(u) = \left|\sin 2(\varphi + \frac{\pi}{4})\sin(\varphi + \frac{\pi}{4} + arctg\ e^{-u})\right|$,

$$\varphi = -\frac{\pi}{4} - arctg\left[2\sqrt{\frac{p}{3}}\cos\frac{y}{3}\right], \cos y = \frac{q}{2\sqrt{\left(\frac{p}{3}\right)^3}},$$

$$p = 2 + \frac{4}{3}e^{-2u},\ q = \frac{e^{-u}}{3} + \frac{16}{27}e^{-3u}.$$

We select from the group of error components expressions (37) the coefficients that determine the nature of the changes of the maximum values of these components:

$$\begin{aligned}
\upsilon_1(u) &= l_2(u) e^{\frac{3u}{2}}, \\
\upsilon_2(u) &= -l_1(u)\sqrt{chu} = \nu_4(u), \\
\upsilon_3(u) &= -l_1(u)\sqrt{chu}\cdot m(u) = \upsilon_2(u)\cdot m(u).
\end{aligned} \tag{38}$$

Dependence of the coefficients (38) on information surveillance time when changing T_H from 0 to T_0 with steps 0,005 T_0 is given in the form of graphs (Fig. 2). As follows from these relationships, the coefficients increase with the increase of information surveillance time.

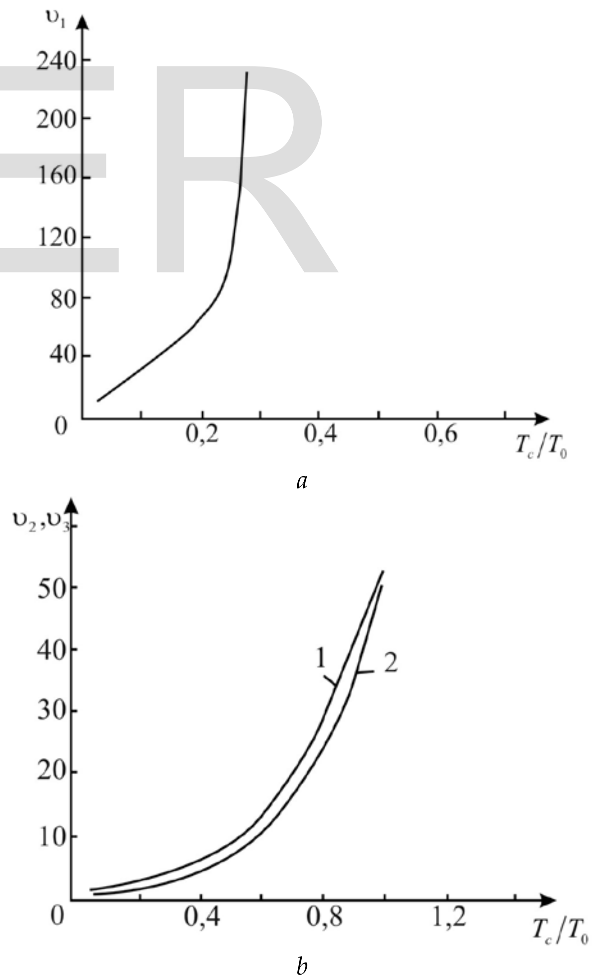

Fig. 2. Dependence of the impact of errors ν_1(a), ν_2 ν_3 (b) on T_c (1-ν_2; 2-ν_3)

ISSN 2229-5518

According to these dependences of the coefficients, dependences of maximum values of error components (Fig. 3 a, b) have been made. Based on these charts, the following conclusions can be made:

- for the zone of the initial conditions (30) the maximum error value $\Delta\hat{R}_1^S$ increases when information surveillance time increases and is determined by the parameter A. So, if A=1° and T_c=0,3 T_0 $\max(\Delta\hat{R}_1^S) = 1{,}33$ Arcsecond (Fig. 3, a). According to (37), based on a cubic dependence $\max(\Delta\hat{R}_1^S)$ on A, by decreasing the radius of the zone of initial conditions arbitrarily small impact of this component on the total evaluation error at the information surveillance time that does not exceed 0,3 T_0 can be achieved;

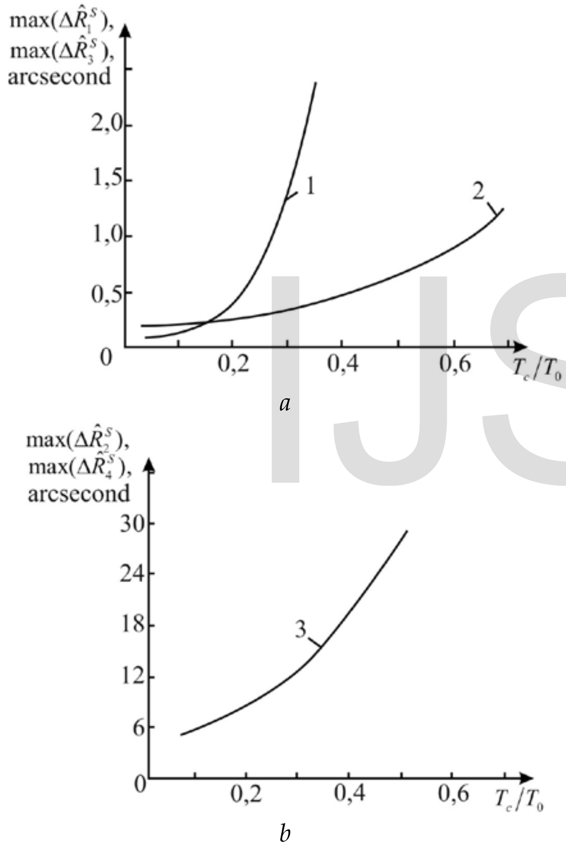

Fig. 3. Dependence of errors $\left|\Delta R_i^S\right|$ on T_c where i: 1 - 1; 2 - 3; 3 - 2; 4

- the maximum values of evaluation errors $\Delta\hat{R}_2^S$ and $\Delta\hat{R}_4^S$ are going up with increasing of information surveillance time. Taking into account that in real devices $(\Delta\omega)\omega_0^{-1} \le 5\cdot10^{-4}$ and $\left|\xi_1\right|\cdot\omega_0^{-1} = 10^{-4}$ when A = 1 ° and T=0,3 T_0 we obtain that $\max(\Delta\hat{R}_2^S) = 1{,}2\,Arcsecond$ and $\max(\Delta\hat{R}_4^S) = 7\,Arcsecond$. Based on the linear dependence of errors on ρ, the errors can be reduced to any small values by choosing ρ;

- if ρ≤1° for information surveillance time T_H=0,3 T_0, the impact of the component $\Delta\hat{R}_3^S$ on the total error $\Delta\hat{R}^S$ is negligibly small (see Fig. 3).

On the other hand, according to (33) ... (36), in the case of preassigned T_c for each evaluation error component there are the optimal polar angles, in the sense of transformation of this component into zero.

For evaluation error $\Delta\hat{R}_1^S = 0$ the optimal polar angle is

$$\varphi_{\text{мпТ}} = k\pi - \frac{\pi}{4}, \tag{39}$$

k = 0; ± 1; ± 2 … .

Evaluation error component $\Delta\hat{R}_2^S$ becomes zero at

$$\varphi_{\text{мпТ}} = k\pi - \frac{\pi}{4} + arctg\ e^{-u}, \tag{40}$$

k=0; ± 1; ± 2 … .

Evaluation error component $\Delta\hat{R}_3^S = 0$ at

$$\varphi_{\text{мпТ}} = k\pi - \frac{\pi}{4} \ \text{i}\ \varphi_{\text{мпТ}} = k\pi - \frac{\pi}{4} - arctg\ e^{-u}, \tag{41}$$

k=0; ± 1; ± 2 … .

Evaluation error component becomes zero at

$$\varphi_{\text{мпТ}} = k\pi - \frac{\pi}{4} - arctg\ e^{-u}, \tag{42}$$

k=0; ± 1; ± 2 … .

In (39) ... (42) the following denotations are accepted

$$u = \omega_0 T_H,\ tg\varphi = \frac{\dot{\alpha}_0}{\omega_0\alpha_0}. \tag{43}$$

Consider the impact of the typical distortions on the accuracy of evaluation of DTG state for the observed law of motion of SE .

1. Linear drift of SE equilibrium, as a result of harmful moments in the case of southern orientation of SE, leads to the error of stable equilibrium (Fig. 4).

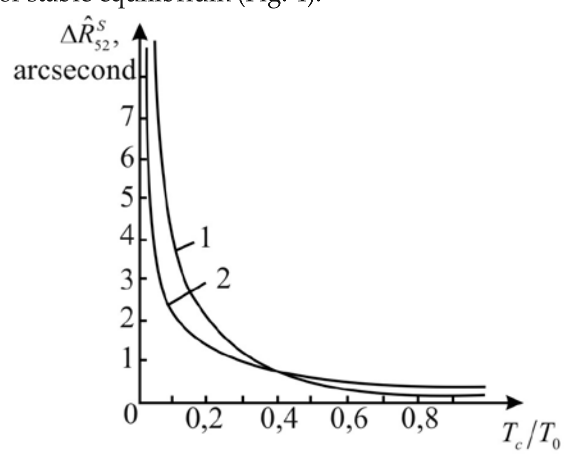

Figure 4 . Dependence of $\Delta\hat{R}_{52}^S$ on such distortions as $e^{-\frac{t}{T}}$ and T_c (a_e=1') at τ, s: 1- 300; 2 – 100

ISSN 2229-5518

2. As it is proved in [9], SE is affected by the moments with a period comparable or much smaller than the period of precession oscillations. In both cases the distortion is given by the expression

$$\Delta\hat{R}_{53}^{N}=a_r d_{33}(\mu,u)\sin\left(\frac{\mu u}{2}+\varphi_1\right), \qquad (44)$$

After substituting (44) in the expression of evaluation error $\Delta\hat{R}_5^S$ and limiting transition at $\Delta t\to 0$, T_c=const,we can write an expression of evaluation error

$$\Delta\hat{R}_{53}^{S}=a_r l_3(\mu,u)\sin\left(\frac{\mu u}{2}+\varphi_1\right), \qquad (45)$$

where

$$l_3(\mu,u)=\frac{2}{\mu}\cdot\frac{\left(\frac{e^u+1}{2}+\frac{ue^u}{e^u-1}\right)\sin\frac{\mu u}{2}-\frac{2}{\mu^2+1}\left[(e^u-1)\cos\frac{\mu u}{2}+\mu(e^u+1)\sin\frac{\mu u}{2}\right]}{u\left(\frac{e^u+1}{2}+\frac{ue^u}{e^u-1}\right)-2(e^u-1)}.$$

Graphical dependency of the coefficient $l_3(\mu,u)$ for μ=2, 3, 5 is shown in Fig. 5. We can see that $l_3(\mu,u)$ reduces from a certain value, which depends on μ, to zero, and then repeatedly crosses the x-axis. Graphical dependency of errors (Fig. 6) shows that the state evaluation error $\Delta\hat{R}_{53}^{S}$, as well as the coefficient $l_3(\mu,u)$, varies according to the harmonic law. Low-frequency disturbance at $T_c\le 0{,}3T_0$ is weakly subdued by the state evaluation algorithm. The increase in information surveillance time $T_c>0{,}15T_0$ and interference frequency reduces the evaluation error. If μ=100 and $T_c\le 0{,}1T_0$, the interference amplitude is hundreds of times smaller than the evaluation error; thus there has been increasing interference that under these conditions is greater when the noise frequency is higher. With increasing $T_c\ge 0{,}15T_0$ the error amplitude is subdued in hundreds of times (Fig. 6, a) and at $T_c\ge 0{,}2$-$0{,}3T_0$ the high frequency harmonic distortion $\Omega\ge 100\omega_0$ hardly causes the evaluation error.

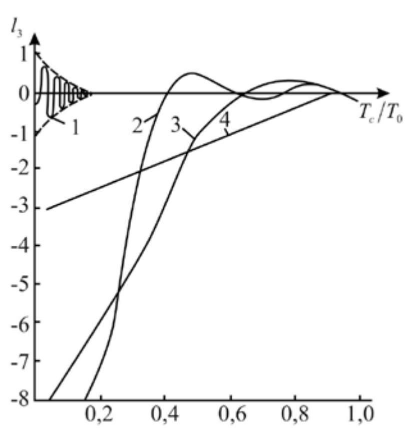

Fig. 5. Dependence of the impact coefficient l_3 on T_c, when μ: 1 – 100; 2 – 5; 3 – 3; 4 – 2

Thus, the low-frequency distortions in the SE motion law at $0{,}1T_0 \le T_c \le T_0$ are weakly subdued by the state evaluation algorithm, and high-frequency distortions (Ω=100ω_0) are effectively filtered at $T_c\ge 0{,}15T_0$, and at T_c=0,2…0,3T_0 they barely cause the evaluation error.

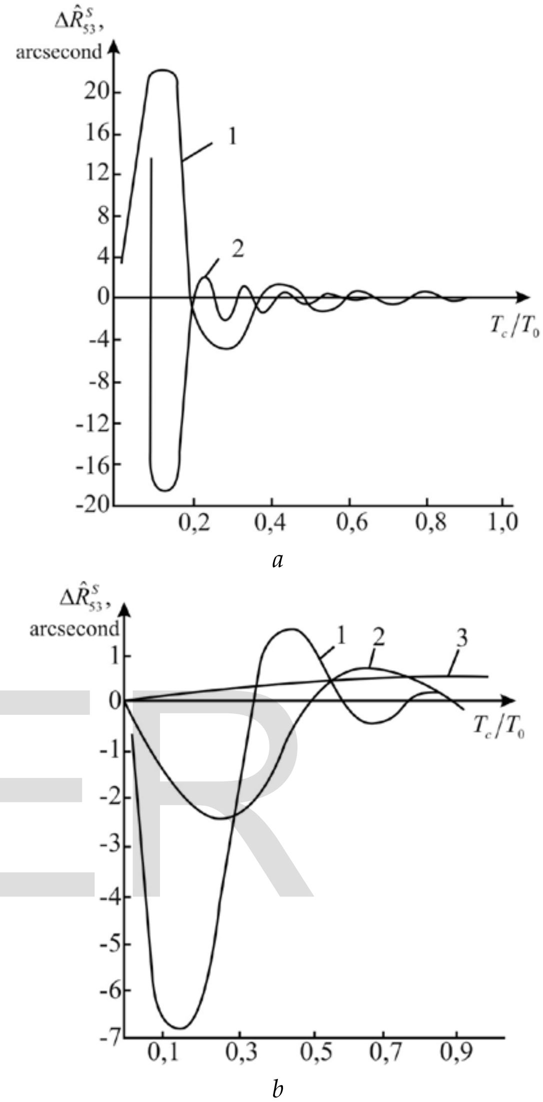

Fig. 6. Dependence of error $\Delta\hat{R}_{53}^{S}$ on distortions such as sinΩt, T_c (a_r=1’), when μ: a – 10 and 5 (respectively curves 1 and 2); b - 3; 2 and 0.5 (respectively curves 1, 2 and 3)

3. If the analysis of evaluation error because of the noise meter (AS) in the observed law of motion of SE as well as in [9], we assume that measuring error is given in the form of white noise and for evaluation error we use the Holder inequality. Under these conditions evaluation can be represented by the expression

$$\left|\Delta\hat{R}_{54}^{S}\right|\le l_{34}(u)\Phi_{\varepsilon}^{S}(T_H), \qquad (46)$$

where

$$l_{34}(u)=\sqrt{\frac{u(u+shu)}{u(u+shu)-8sh^2u\cdot 2^{-1}}};\ \Phi_{\varepsilon}^{S}(T_c)=\sqrt{T_H^{-1}\int_0^{T_c}\varepsilon^2(t)dt};$$

$\Phi_{\varepsilon}^{S}(T_c)$ is quadratic mean value of the random function on the interval [0 , T].

Just as in the previous case, the coefficient $l_{34}(u)$ decreases with increasing of T_c and actually does not change at $T_c \geq 0{,}2$-$0{,}3\,T_0$, and further increase in information surveillance time is unnecessary (Fig. 7).

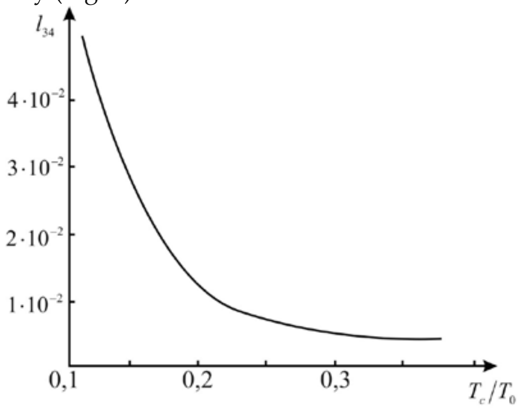

Fig. 7. Dependence of the impact coefficient l_{34} on T_c

Conclusions:

As the result of the research done in this paper, the state evaluation algorithm of unstable equilibrium of DTG was developed. Evaluation errors were examined.

The analysis of mathematical expressions of evaluation errors revealed that the evaluation error contains five components, based on:

- non-linear motion trajectory distortions of the SE in large deviations from the North-South flat;
- non-linear frequency deviations of the precession oscillations, which occur due to anisochronism of movements of the SE near the unstable equilibrium;
- the SE motion trajectory bending due to the moments such as viscous friction;
- the precession oscillations cyclic frequency divergency that is used in the state evaluation algorithm, and in DTG precession oscillations frequency algorithm;
- presence of the disturbance such as linear and exponential drift of equilibrium state of the SE, harmonic interferences and white noise interferences in the observable motion law.

The first two evaluation errors components are directly proportional to the cube of polar radius $\rho = A = \sqrt{\alpha_0^2 + (\dot{\alpha}_0\omega^{-1})^2}$, where α_0, $\dot{\alpha}_0$ is the initial angle and angular velocity of the SE movement in the respect of the unstable equilibrium. Another two components are proportional to the polar radius. Optimal relations (in terms of evaluation error vanishing) between the information surveillance time and initial conditions $\alpha_0 = const$, $\dot{\alpha}_0 \to 0$ were found. It is demonstrated that all of the enumerated components of the evaluation error grow with the increase of the information surveillance time, when initial conditions are close to $\alpha_0 = const$ and $\dot{\alpha}_0 \to 0$.

The examinations of the disturbance influence on the evaluation precision show that:

- disturbance such as linear drift of equilibrium state of the SE causes errors proportional to the drift speed and information surveillance time;
- exponential drift disturbance causes the error, which is monotone-decreasing, when information surveillance time increases. The smaller the time constant of the exponent is, the faster the error decrease is;
- low-frequency harmonic interference, the frequency of which is commensurable with the precession oscillations frequency, is weakly eliminated by the state evaluation algorithm;
- high-frequency harmonic interference, the frequency of which 100 times exceeds the precession oscillations frequency, and random white noise interference are effectively filtered by the state evaluation algorithm. Information surveillance time needed for complete HF-interferences and white noise interferences elimination has to be 0.2 … 0.3 of the precession oscillation period.

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